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PRICING POLICIES CONTINGENT
ON OBSERVED PRODUCT QUALITY

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ABSTRACT

Expanded use of incentive contracts has created interest in procurement arrangements in which unit purchase price varies as a function of observed product quality. Under the assumption that at a known cost, a producer can control the true quality of his output, a production and procurement situation is described in which a risk-averse producer and consumer both attempt to maximize expected profit --the consumer by selecting a pricing strategy and sample size, and the producer by then selecting the product quality.

Let product quality be denoted by p . The consumer will desire the selected product quality to be some p^* . If at a fixed sample size, n , a price schedule exists which is acceptable to the risk-averse producer and also maximizes producer expected profit at p^* , this price schedule is a "motivating" price schedule. For fixed (n, p) , a motivating price schedule must be the solution to a specified linear programming problem. It will be a piecewise constant function of the observed quality and need have no more than six distinct price levels.

A method of deriving continuous, piecewise-linear price schedules is briefly described, and extensions of the basic approach are noted.

PRICING POLICIES CONTINGENT ON OBSERVED PRODUCT QUALITY*

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1. INTRODUCTION

Expanded use of incentive contracts by government and industry has created interest in procurement arrangements which provide that the payment to a producer will vary with the degree of product conformance to design specification as measured by a sample or test of the product. Johns and Lieberman⁽¹⁾ describe a situation in which a customer purchases batches of N items, draws a sample of n ($n \leq N$) from each batch, and then pays a unit price, $\varphi(x)$, based on x , the number of defective items discovered in the sample. Flehinger and Miller⁽²⁾ postulate a situation in which after agreeing to purchase equipment with a specified failure rate, the customer records the number of observed failures, x , in a prespecified test interval, t , and pays the manufacturer a premium, $\varphi(x)$.

In this paper, attention is restricted to the production situation in which at a cost $h(p)$, the producer can control p , the probability that any item produced is defective. The cost $h(p)$ is known to both the producer and consumer and includes a unit profit.

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The model reported here is based on a portion of a dissertation submitted to the Committee on Operations Research, Stanford University, in partial fulfillment of the requirements for the degree of Doctor of Philosophy. The author is indebted to Professor Gerald J. Lieberman for his many valuable suggestions. The work was supported in part by Office of Naval Research Contract Nonr-225(53).

Items produced are formed into batches of N , and the consumer stipulates in the contract a sample size, n , and payment schedule, $\varphi(x)$, based on the number of defectives discovered in the sample. The consumer is charged c per unit inspected.

Previous investigations of this and similar situations have assumed that the producer will accept any price schedule promising a non-negative expected profit at some quality level, p , and that he will select that p which maximizes expected profit. It is usually assumed further that once the consumer knows p^* , the quality level he desires, he will accept any pricing arrangement which both induces the producer to select p^* and minimizes consumer expected outlay plus sampling cost at p^* .

For a detailed discussion of pricing policies that result from only these assumptions see ⁽³⁾. Two results are particularly pertinent. If payments, $\varphi(x)$, are not bounded above and below, a motivating price schedule can always be found using a sample of only 1. If $\varphi(x)$ is bounded below, say by $m < h(p^*)$, the sample size required for a motivating price schedule is reasonably large from an operational point of view. However, except in special cases, the optimal price schedule calls for payment only if zero defectives are observed in the sample, and provides only the minimum payment, m , otherwise. While such schedules achieve the minimum sample size required for a motivating policy, they are unsatisfactory in practice. Optimal "one-chance" policies of this type provide no opportunity to control the risk of incorrect payment due to sampling variation. Furthermore, in order to increase the probability of positive net return

to an acceptable value, the risk-averse producer may decide that a quality level is required far superior to that desired by the consumer. If the expected profit at this producer-selected level is not adequate, the producer will reject the contract offer. To deal with these considerations directly, a model must recognize that both producer and consumer actually maximize expected utility rather than expected profit.

A tactic used in constructing acceptance sampling plans to avoid postulating complete utility functions for both consumer and producer is to focus directly on the probabilities of incorrect decisions at relevant quality levels. Analogous reasoning in the contingent pricing case leads to the assumption that the producer desires protection against underpayment if he selects p^* , that quality implicitly directed by the consumer's knowledge of $h(p)$ and his subsequent choice of $\varphi(x)$ and n . It is assumed that the producer can state the minimum unit price, w , which he desires assurance of receiving if true quality is as good as p^* . Since even with acceptable quality ($p \leq p^*$) any batch may yield a sample containing a disproportionately high number of defectives, the producer will accept a contingent pricing policy which given n and $p \leq p^*$ promises payment less than w with frequency less than some specified α . Similarly it is assumed that the consumer can state v , the maximum unit price he is willing to pay when quality is as poor as p_b , and β , the relative frequency with which overpayment will be permissible. For several values of p , a consumer may also be able to estimate $V(p)$, the expected unit value of each item produced. In this paper, $V(p)$ is assumed to exist and to be concave

and decreasing, although as will be seen, the existence of motivating price schedules is not affected by the existence and knowledge of $V(p)$. Actual payments, $\varphi(x)$, are assumed to be bounded above by M , below by m , and to be monotonically decreasing in x . The production cost function, $h(p)$, is assumed strictly convex and decreasing in the quality interval of interest, and lot formation and sampling are assumed to take place in such a manner that x is a binomial random variable.

2. BASIC MODEL

The situation described in Section 1 is formally a constrained, two person non-zero-sum game in which the strategy of one player (the pricing strategy of the consumer) is revealed to the other player (the producer) in advance of his choice of a strategy. This type of game has a well defined notion of a solution. The consumer knows the producer will select a quality level which will net the producer the maximum expected return under the pricing policy chosen by the consumer. The consumer must, therefore, select the pricing policy which will net him the greatest return under the producer's corresponding optimal strategy.

Let $\pi_x(n, p) = \binom{n}{x} p^x q^{n-x}$, where $q = 1 - p$,

$\delta_x(n, p) = d/dp \pi_x(n, p)$,

$g(p)$ = expected value of $\varphi(x)$ given p for fixed n ,

$T_c(n, \varphi, p)$ = consumer expected profit and,

$T_p(n, \varphi, p)$ = producer expected profit.

The consumer and producer expected net gains are respectively

$$T_c(n, \varphi, p) = N V(p) - N g(p) - nc, \quad (2.1)$$

$$T_p(n, \varphi, p) = N g(p) - N h(p), \quad (2.2)$$

and the preceding assumptions imply that no contract will be acceptable to both parties unless the conditions

$$\text{pr} [\varphi(x) \geq w \mid p \leq p^*] \geq 1 - \alpha, \quad (2.3)$$

$$\text{pr} [\varphi(x) \leq v \mid p \geq p_b] \geq 1 - \beta, \quad (2.4)$$

$$M \geq \varphi(0) \geq \dots \geq \varphi(n) \geq m, \quad (2.5)$$

$$N g(p^*) - N h(p^*) \geq 0, \quad (2.6)$$

are satisfied. The consumer seeks to maximize expected profit, (2.1), by choosing $n, \varphi_0, \dots, \varphi_n$, which satisfy (2.3)-(2.6) knowing that the producer will choose the p which maximizes (2.2).

Theorem 2.1. With n and p fixed, say at (n, p^*) where $0 < p^* < 1$, it is necessary that a set of prices, φ_x , maximizing (2.1) while simultaneously maximizing (2.2) at p^* and satisfying (2.3)-(2.6) must also minimize

$$\sum_{x=0}^n \pi_x(n, p^*) \varphi_x \quad (2.7)$$

and satisfy

$$\sum_{x=0}^n \delta_x(n, p^*) \varphi_x = h'(p^*), \quad (2.8)$$

$$\varphi_{k_\alpha} \geq w, \quad (2.9)$$

$$\varphi_{k_\beta} \leq v, \quad (2.10)$$

$$M \geq \varphi_0 \geq \dots \geq \varphi_n \geq m, \quad (2.11)$$

$$\sum_{x=0}^n \pi_x(n, p^*) \varphi_x \geq h(p^*), \quad (2.12)$$

where k_α and k_β are described in the proof.

Proof. From (2.1) maximizing T_c at (n, p^*) is equivalent to minimizing $g(p^*)$ by choosing $\varphi_0, \dots, \varphi_n$, yielding (2.7) as the expression to be minimized.

A condition necessary for T_p to be a maximum at p^* , $0 < p^* < 1$,

$$\text{is } d/dp [g(p) - h(p)] = 0 \text{ at } p = p^* \text{ or } \sum_{x=0}^n \delta_x(n, p^*) \varphi_x = h'(p^*)$$

which is (2.8). If after deriving a set of φ_x satisfying (2.7)-(2.12) it can be verified (perhaps graphically) that $g(p) - h(p)$ is maximized at p^* , it is then also true that the solution to the linear programming problem (2.7)-(2.12) is a solution to (2.1)-(2.6).

Constraints (2.5) and (2.6) are already linear in the φ_x .

Constraint (2.3) requires specifically

$$\text{pr} [\varphi(x) \geq w \mid p = p^*] \geq 1 - \alpha, \quad (2.13)$$

which is equivalent to

$$\sum_{k=0}^n \text{pr} [\varphi(k) \geq w] \pi_k(n, p^*) \geq 1 - \alpha. \quad (2.14)$$

Let k'' be the greatest integer for which $\varphi_k \geq w$. From (2.14) it is necessary that

$$\sum_{k=0}^{k''} \pi_k(n, p^*) \geq 1 - \alpha.$$

Let k_α denote the smallest integer such that

$$\sum_{k=0}^{k_\alpha} \pi_x(n, p^*) \geq 1 - \alpha.$$

In order for (2.14) to be satisfied, it is necessary that $k_\alpha \leq k''$, and since (2.11) requires $\varphi(x)$ to be monotonically decreasing, $\varphi(k_\alpha)$ cannot be less than w . Thus (2.3) requires (2.9). It is also true that (2.9) with k_α defined above, implies (2.13). And further, it is easily shown that (2.13) implies (2.3) since for $j \leq n$,

$$\frac{d}{dp} \sum_{k=0}^{k=j} \pi_x(n, p) = \frac{-n}{j!(n-j-1)!} p^j q^{n-j-1}$$

which is non-positive. Hence $p < p^*$ implies

$$\sum_{k=0}^{k_\alpha} \pi_x(n, p) \geq \sum_{k=0}^{k_\alpha} \pi_x(n, p^*),$$

and if (2.13) is satisfied, then (2.3) is satisfied. Thus (2.9) is both a necessary and sufficient condition for (2.3). In a similar fashion, it can be verified that (2.4) implies (2.10) where k_β is the greatest integer for which

$$\sum_{x=k_\beta}^n \pi_x(n, p_\beta) \geq 1 - \beta \quad (2.15)$$

and also that (2.10) with k_β defined by (2.15) implies (2.4). This completes the proof that with n and p fixed, solutions to (2.1)-(2.6) are solutions to (2.7)-(2.12).

The inclusion of probabilistic safeguards against the risk of incorrect payment due to sampling variation determines a minimum

sample size at which discrimination between p^* and p_b can take place with the required confidence since in any case of interest $w > v$ and by the monotonicity of $\varphi(x)$, k_α must be strictly less than k_β . In addition, the choice of w and v constrain the values of the price function $\varphi(x)$.

The transformation

$$y_j = \varphi_j - \varphi_{j+1}, \quad j = 0, 1, \dots, n-1 \quad (2.16)$$

$$y_n = \varphi_n - m, \quad (2.17)$$

yields a linear programming problem equivalent to (2.7)-(2.12), but containing only 5 constraints. From (2.16) and (2.17)

$$\varphi_j = m + \sum_{i=j}^{n-1} y_i.$$

Choosing φ_x to minimize (2.7) is equivalent to choosing y_i to minimize

$$m + \sum_{i=0}^n \sum_{x=0}^i y_i \pi_x.$$

$$\text{Let } \prod_i = \sum_{x=0}^i \pi_x(n, p^*) \text{ and } \Delta_i = \sum_{x=0}^i \delta_x(n, p^*).$$

The problem (2.7)-(2.12) becomes that of choosing $y_i \geq 0$ to minimize

$$\sum_{i=0}^n \prod_i y_i \quad (2.18)$$

subject to the constraints

$$\sum_{i=0}^n \Delta_i y_i = h'(p^*), \quad (2.19)$$

$$\sum_{i=k_\alpha}^n y_i \geq w - m, \quad (2.20)$$

$$\sum_{i=k_\beta}^n y_i \leq v - m, \quad (2.21)$$

$$\sum_{i=0}^n y_i \leq M - m, \quad (2.22)$$

$$\sum_{i=0}^n \prod_i y_i \geq h(p^*) - m. \quad (2.23)$$

The computing time required to solve a linear programming problem increases much more rapidly with the number of constraints than with the number of variables. Thus while the original linear programming problem can be solved either by including the monotonicity constraints explicitly, or by using a bounded variables algorithm, the equivalent problem (2.18)-(2.23) is solved much more rapidly. Furthermore, it is apparent from this formulation that since at most five y_i need be positive, there need be at most six distinct price levels, p_x , in an acceptable motivating strategy.

3. CONSUMER SELECTION OF p .

The production cost function, $h(p)$, is taken to be known and fixed, but the parameters $M, m, \alpha, \beta, w, v$, and p_b are essentially arbitrary.

Given n and p , a price strategy, φ_x , exists if the problem (2.7)-(2.12) has a feasible solution. Analytic conditions sufficient for the existence of a solution can be stated* but in practice, if α , β , and p_b are selected realistically, and the minimum sample size is determined at which the appropriate discrimination between p^* and p_b can be achieved, one then desires that a motivating policy exist. An acceptable policy can always be made to exist by increasing $(M - m)$ or relaxing the price restrictions w and v sufficiently.

If once selected, the parameters $M, m, w, v, \alpha, \beta$, and p_b are fixed, the problem is not simply that of determining the pricing policy which minimizes total expected cost at some prespecified n and p , but rather that of determining the (n^*, p^*) which maximizes

$$\left\{ NV(p) - nc - \min_{\varphi} \sum_{x=0}^n \pi_x(n, p) \varphi_x \right\}$$

where φ_x satisfies (2.8)-(2.12). The bracketed quantity is the function $T_c(n, \varphi, p)$ and the constraints (2.8)-(2.12) describe the set of points (n, p) on which $T_c(n, \varphi, p)$ is defined. Very little can be said in general about the concavity of $T_c(n, \varphi, p)$ and the convexity of the region on which it is defined. If $V(p)$ is known for only a few values of p , a pricing policy and sample size may be determined for each relevant p by finding the minimum n for which $k_\alpha(p) < k_\beta(p_b)$ and determining $T_c(n, \varphi, p)$ for that (n, p) . If $g(p) = h(p)$, this minimum n is optimal for the current p . If $g(p) > h(p)$, increasing n may cause $g(p)$ to decrease toward $h(p)$ yielding increased profit to the consumer. However, it is not true that $g(p)$ need decrease monotonically to $h(p)$ as n increases, and therefore a search procedure

* See Ref. 3, pp. 74-80.

may not yield the optimal sample size for a given p .

In seeking optimal pricing strategies using representative data, a generalized Fibonacci search was used. The Fibonacci search generalized to two variables requires that when searching in the variable n , $T_c(n, \varphi, p)$ is evaluated at $p^*(n)$, where $p^*(n)$ maximizes $T_c(n, \varphi, p)$ for fixed n . ⁽⁴⁾ At each search point (n, p) the linear programming problem (2.7)-(2.12) is solved. This evaluation procedure was programmed in Fortran for the IBM 7090 at Stanford University. Using a ten point search over the sample sizes, and a ten point search over the quality interval $(0, p_b)$, the procedure may be viewed as 100 sequential evaluations to determine the maximum of a function on a 143×89 lattice. Solutions were almost always obtained in less than 30 seconds. Of course, unimodality of the function $T_c(n, \varphi, p)$ must be verified to show that any particular strategy is optimal.

Example 1.

Using the following data and functions,

$$\begin{aligned}h(p) &= .4 - .2p - .01 \exp(.05/p), \\N &= 200, M = 1, m = .15, c = .172, \\w &= .75h(p), v = .45V(p), p_b = .175, \\\alpha &= .05, \theta = .10, V(p) = 1 - 3p,\end{aligned}$$

an optimal contingent pricing policy is found at $n^* = 39$, $p^* = .0325$, and is shown in Figure 3.1. Figure 3.2 and Table 3.1 present the expected payments and profit resulting from this policy.

4. PIECEWISE LINEAR PRICE SCHEDULES.

Current contingent price schedules are usually piecewise linear. In most cases, payments are determined by negotiating the sample size, maximum and minimum payments, and the number of defectives at which the maximum and minimum payments occur. Such policies can be written

$$\varphi_x = \varphi_0 \quad 0 \leq x \leq a \quad (4.1)$$

$$\varphi_x = \left[\frac{b-x}{b-a} \right] \varphi_0 + \left[\frac{x-a}{b-a} \right] \varphi_n, \quad a < x < b \quad (4.2)$$

$$\varphi_x = \varphi_n, \quad b \leq x \leq n \quad (4.3)$$

and can be derived by replacing constraint (2.11) by

$$M \geq \varphi(x) \geq m, \quad 0 \leq x \leq n \quad (4.4)$$

and (4.1)-(4.3). Although in general, a may range from 0 to $n - 1$, and b may range from $a + 1$ to n , suggesting $n(n + 1)/2$ evaluations for each fixed (n, p) , it can be shown that due to the relations (2.9) and (2.10), the number of candidate endpoints (a, b) leading to feasible policies is very small. By making use of these relations, numerical determination of piecewise linear price schedules is practicable. Computation time for the linear case is greater than that required in the basic model since (a^*, b^*) must be found for each fixed (n, p) , but problems containing 50 search points (n, p) require only about 40 seconds on the IBM 7090. The most efficient procedure is to solve the problem for a basic solution and then use the resulting

sample size as a lower bound in the linear case, thus reducing the number of sample sizes to be searched through.

Example 2.

For the data,

$$h(p) = .25 - .415p - .06hp,$$

$$N = 500, M = .45, m = .10, c = .12,$$

$$w = h(p), v = .4V(p), p_b = .17,$$

$$V(p) = 1 - 1.5p, \alpha = .25, \beta = .25,$$

an optimal basic policy is found at $n = 36$, and $p = .052$. The piecewise linear pricing policy requires a larger sample size and is found at $n = 46$, $p = .052$. These policies are shown in Figure 3.3. Figure 3.4 displays the expected payments resulting from both policies, and Table 3.2 contains the producer expected profit under both policies.

Consumer unit expected profit using the linear policy is .517 while with the less restricted basic policy, the consumer unit profit is .5194. In general, the use of piecewise linear pricing schedules requires larger sample sizes, and hence yields lower consumer unit profits. From Figure 3.4 it can be noted that the basic policy does not provide the same discrimination in expected payment as does the piecewise linear policy, but both policies satisfy conditions (2.3) and (2.4). In any situation where the producer would prefer a linear price schedule, the consumer must decide whether it is advantageous to offer an increased unit expected profit in return

for the producer acceptance of a piecewise constant price schedule.

5. EXTENSIONS.

The assumptions introduced in section 1 can be weakened to include a more general class of production and procurement situations. When testing or inspection is non-destructive, the producer will usually replace all defectives discovered during the inspection process at no additional cost to the consumer. Assume that the cost to the producer of replacing discovered defectives is the production cost, $h(p)$, plus a fixed charge r , reflecting additional transportation, testing, and handling costs. The expected net gains (2.1) and (2.2) are then replaced by

$$T_c(n, \varphi, p) = (N - n) V(p) + n [V(0) - c] - Ng(p). \quad (5.1)$$

$$T_p(n, \varphi, p) = Ng(p) - Nh(p) - npr, \quad (5.2)$$

and (2.8) is appropriately modified. The motivating policy, φ_x , is the solution to a linear programming problem as before.

If the producer does not select p precisely, but instead chooses a mean quality, μ , at a known cost $h(\mu)$, and the resulting distribution of the random variable p is known to be $f(p|\mu, \theta_1, \dots, \theta_k)$ where the θ_i are known, uncontrollable, parameters, we suppose then, that the consumer chooses (n, φ) to maximize expected net gain, and that the producer selects μ to maximize expected profit subject to the assumptions of section 1. This problem is identical to that already developed with $h(p)$ and $h'(p)$ replaced by $h(\mu)$ and $h'(\mu)$, and with

$$\pi_x(n, \mu) = \int_0^1 pr [x|n, p] f(p|\mu) dp \quad (5.3)$$

$$\delta_x(n, \mu) = \partial/\partial\mu \pi_x(n, \mu) \quad (5.4)$$

k_a satisfying $\sum_{j=0}^{k_a} \pi_j(n, \mu)$, and k_b satisfying $\sum_{j=k_b}^n \pi_j(n, p_b)$.

If further, the cost of selecting the mean quality, μ , is not known precisely, but is $h(\mu) + \eta$, where η is a random variable with $E(\eta) = 0$, and known distribution, the essential problem is the same. Constraint (2.6) is replaced by

$$\text{pr } [g(\mu^*) \geq h(\mu^*) + \eta | \mu \leq \mu^*] \geq 1 - \gamma, \quad (5.5)$$

for some γ , $0 \leq \gamma \leq 1$. This is equivalent to a constraint linear in φ_x ,

$$\sum_0^n \pi_x(n, \mu^*) \varphi_x \geq h(\mu^*) + F_{\eta}^{-1}(1 - \gamma) \quad (5.6)$$

where $u = F_{\eta}^{-1}(1 - \gamma)$ is the minimum u such that $F_{\eta}(u) \geq 1 - \gamma$ given $\mu = \mu^*$.

Thus, in situations where it is appropriate to classify items as defective or non-defective, and in which the risk aversion of buyer and seller can be described by their willingness to accept incorrect payments at relevant quality levels, appropriate incentive price schedules can be derived in the manner described. Lifetesting situations can be included in this framework by defining success to be survival beyond a fixed point in time, and continuously varying parameters can be used to index success by specifying an acceptable interval of performance. The derivation of optimal contingent pricing policies for continuously varying parameters will be described in a forthcoming report.

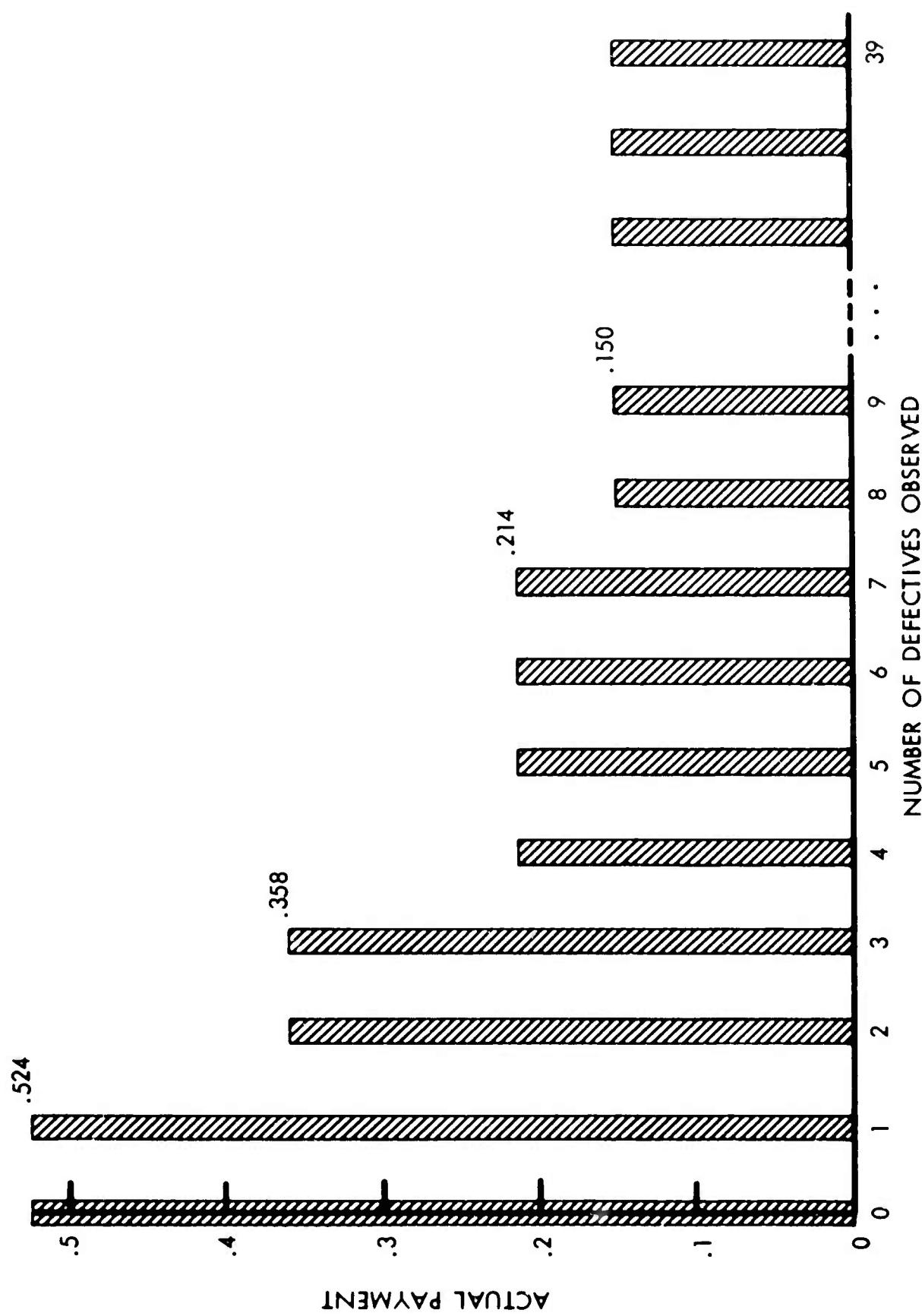


Figure 3.1 -- An Optimal Price Schedule, Example 1

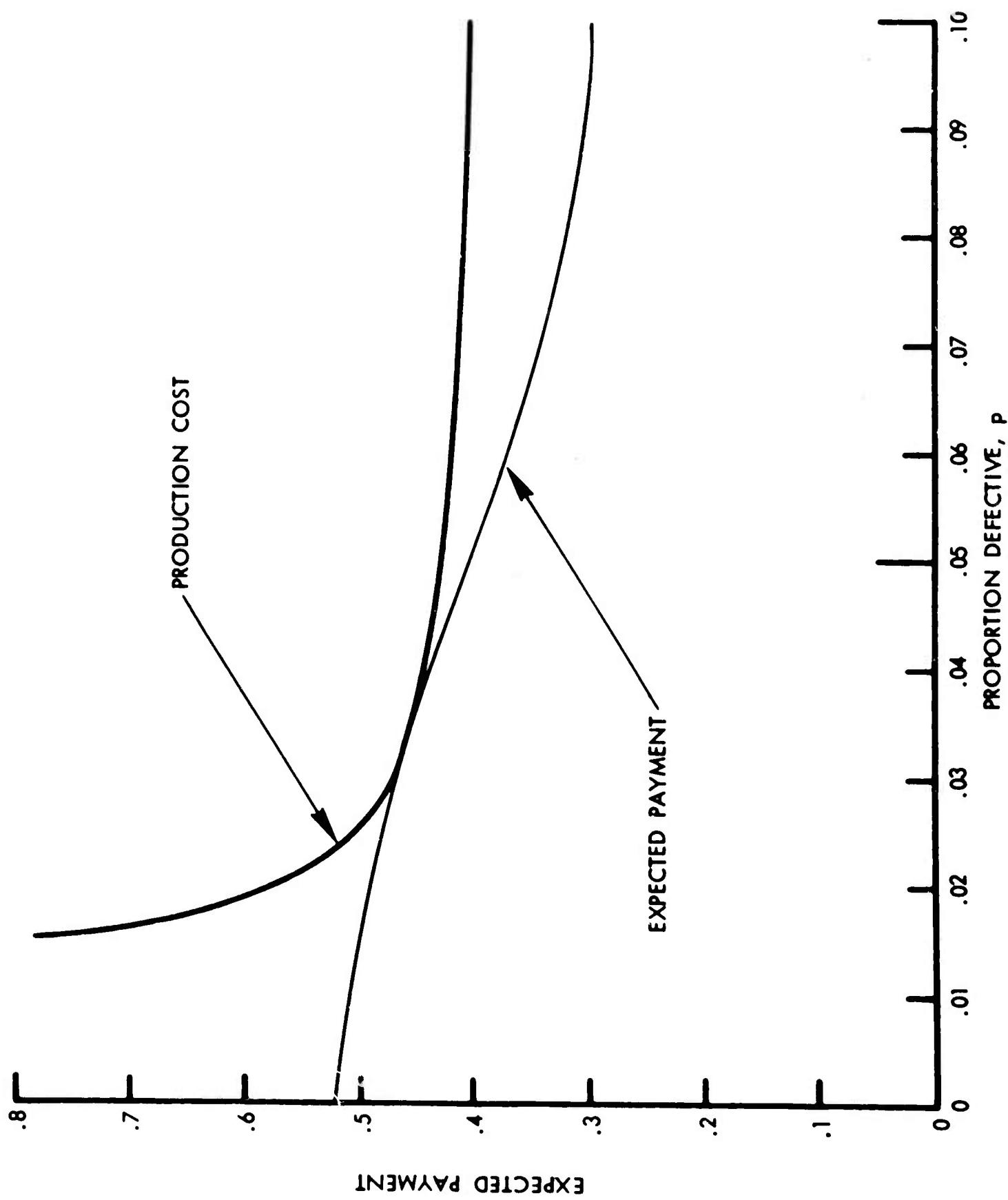


Figure 3.2 -- Expected Payment, Example 1

Table 3.1
Expected Payment and Profits, Example 1

<u>p</u>	<u>Value at p</u>	<u>Cost at p</u>	<u>Expected Payment at p</u>	<u>Producer Profit at p</u>	<u>Consumer Profit at p</u>
.010	.921	2.4758	0.514	-1.961	0.4069
.015	.906	0.7894	0.504	-0.285	0.4019
.020	.891	0.5666	0.492	-0.074	0.3988
.025	.876	0.4984	0.479	-0.019	0.3969
.030	.861	0.4681	0.465	-0.002	0.3957
.0325	.849	0.4544	0.454	0.000	0.3951
.035	.846	0.4514	0.451	-0.000	0.3950
.040	.831	0.4409	0.436	-0.004	0.3944
.045	.816	0.4335	0.422	-0.011	0.3937
.050	.801	0.4281	0.408	-0.019	0.3928

Table 3.2
Expected Payments and Profits, Example 2

<u>p</u>	<u>Cost at p</u>	<u>Expected Payment (basic)</u>	<u>Expected Payment (linear)</u>	<u>Producer Profit (basic)</u>	<u>Producer Profit (linear)</u>
.010	.5037	.4374	.4208	-0.0663	-.0830
.020	.4608	.4312	.4196	-0.0295	-.0411
.030	.4339	.4222	.4159	-0.0117	-.0180
.040	.4137	.4106	.4085	-0.0031	-.0052
.050	.3970	.3969	.3968	-0.0001	-.0002
.052	.3936	.3936	.3940	.0000	.0000
.060	.3827	.3817	.3810	-0.0009	-.0016
.070	.3699	.3656	.3617	-0.0042	-.0082
.080	.3582	.3493	.3397	-0.0089	-.0186
.090	.3475	.3333	.3160	-0.0142	-.0315
.100	.3374	.3178	.2916	-0.0196	-.0458

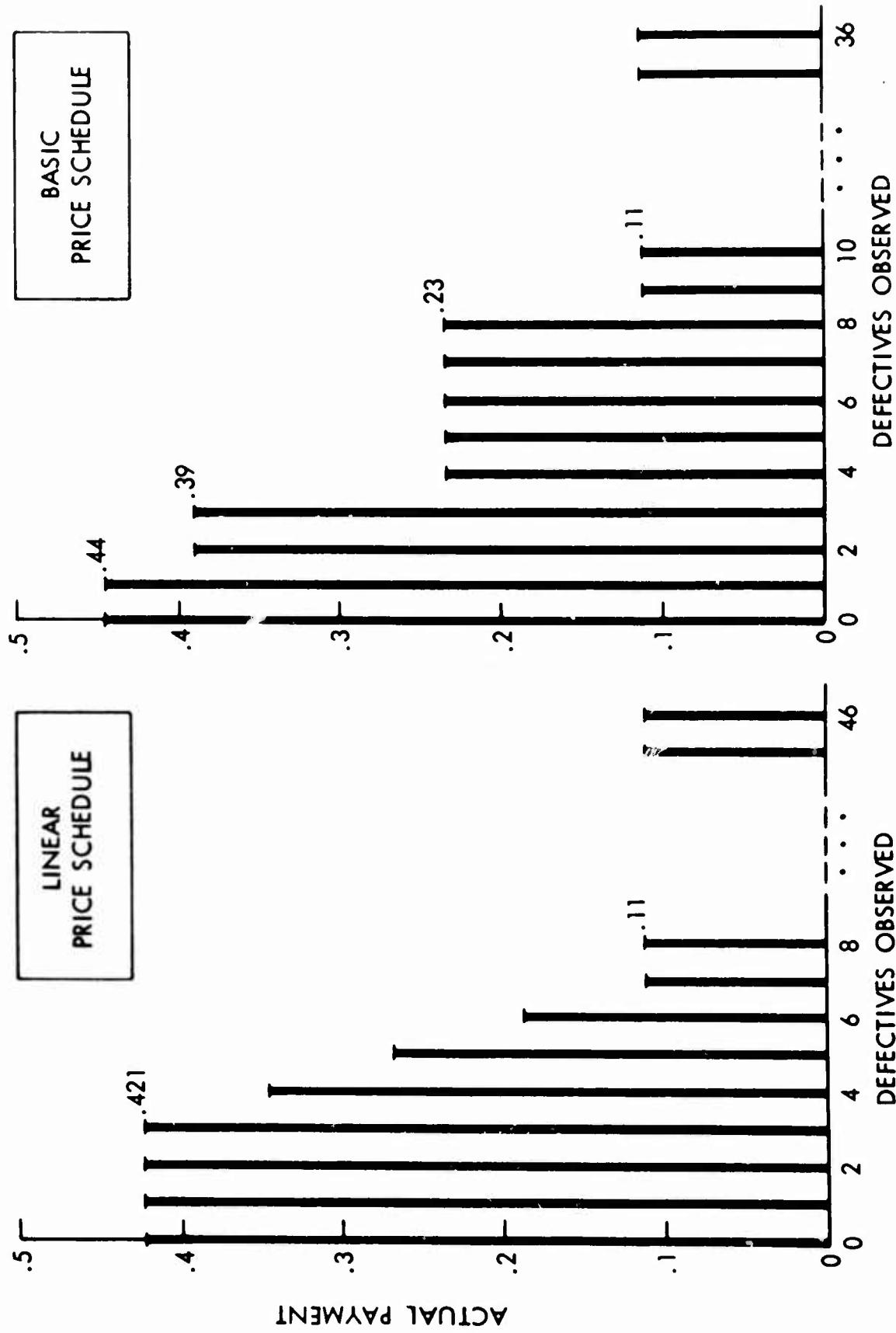


Figure 3.3 -- Optimal Price Schedules, Example 2

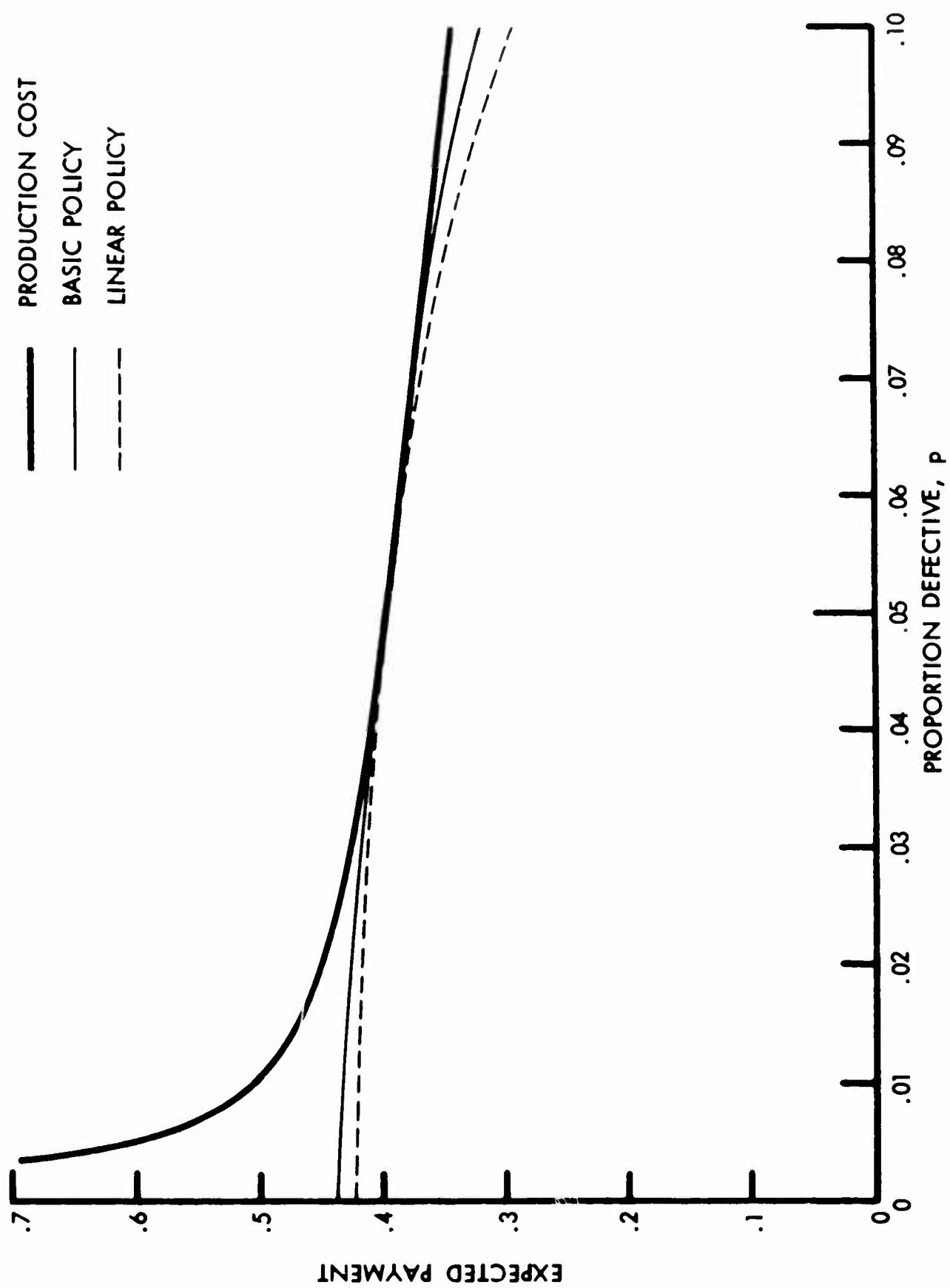


Figure 3.4 -- Expected Payments, Example 2

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